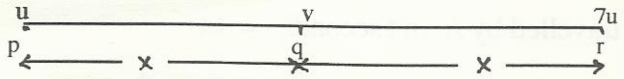
1995

1(a)



(i) Stage pq $v^2 = u^2 + 2fx$ 5

Stage qr $49u^2 = v^2 + 2fx$ 5

$v^2 - u^2 = 49u^2 - v^2$

$v = 5u$ 5 15

(ii) Stage pq $v = u + ft \Rightarrow 5u = u + ft_1$

$\Rightarrow t_1 = \dfrac{4u}{f}$ 5

Stage qr $v = u + ft \Rightarrow 7u = 5u + ft_2$

$\Rightarrow t_2 = \dfrac{2u}{f}$ 5 10

(b) (i) $v^2 = u^2 + 2fs \Rightarrow 0 = u^2 + 2(-g)(3)$ 5

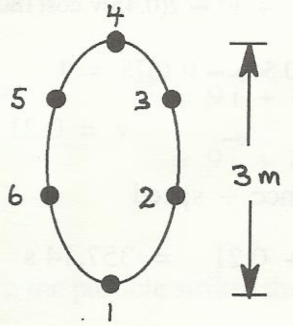
$\Rightarrow u = \sqrt{6g}$ 5 10

use $s = ut + \frac{1}{2}at^2$

(ii) $v = u + ft \Rightarrow 0 = \sqrt{6g} - g(3t)$

$\Rightarrow t = \dfrac{\sqrt{6g}}{3g}$ or 0.26s 5 5

(iii)



$s_6 = \sqrt{6g}.\dfrac{\sqrt{6g}}{3g} - \dfrac{g}{2}.\dfrac{6g}{9g^2}$

$= 2 - 1/3 = 5/3$ 5

$s_5 = \sqrt{6g}.2.\dfrac{\sqrt{6g}}{3g} - \dfrac{g}{2}.\dfrac{4.6g}{9g^2}$

$= 4 - 4/3 = 8/3$ 5 10

$s_6 = s_2 = 5/3$ $s_5 = s_3 = 8/3$ $s_4 = 3$

1995

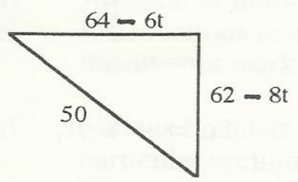
2(a) (i) Distance travelled by A in t seconds $= 6t$

Distance travelled by B in t seconds $= 8t$ — 5

Distance of A from O after t seconds $= 64 - 6t$ — 5

Distance of B from O after t seconds $= 62 - 8t$ — 5 — 15

(ii)



$(64-6t)^2 + (62-8t)^2 = (50)^2$ — 5

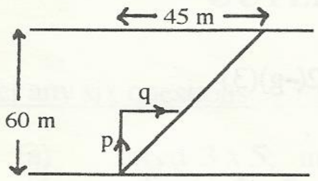
$4096 - 768t + 36t^2 + 3844 - 992t + 64t^2 = 2500$

$100t^2 - 1760t + 5440 = 0$

$(10t - 40)(10t - 136) = 0$

$t = 4$ s or $t = 13.6$ s. — 5 — 10

(b) (i)

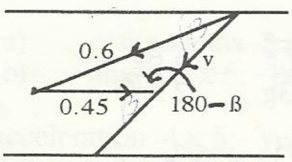


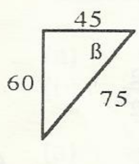
speed = distance ÷ time — 5

horiz. $q = 45 \div 100 = 0.45$ — 5

vert. $p = 60 \div 100 = 0.6$ — 5 — 15

(ii)





$\cos(180-ß) = -\cos ß = -0.6$

$(0.6)^2 = (0.45)^2 + v^2 - 2(0.45)v\cos(180-ß)$

$v^2 + 0.54v - 0.1575 = 0$

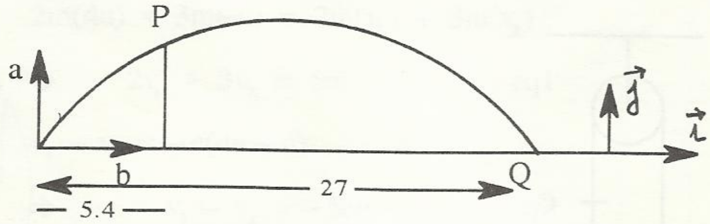
$\Rightarrow \quad v = 0.21$ — 5

time = distance ÷ speed

$= 75 \div 0.21 = 357.14$ s — 5 — 10

1990 1995

3(a)



(i) At Q the $\vec{j}$ component of r = 0

$\Rightarrow$ $a(3) - \frac{g}{2}.(9) = 0$

$\Rightarrow$ $a = \frac{3g}{2}$ or 14.7 — 5

At Q the $\vec{i}$ component of r = 27

$\Rightarrow$ $b(3) = 27$

$\Rightarrow$ $b = 9$ — 5 10

(ii) At P the $\vec{i}$ component of r = 5.4

$\Rightarrow$ $9(t) = 5.4$

$\Rightarrow$ $t = 0.6$ — 5

At P the $\vec{j}$ component of r = 14.7(0.6) − 4.9(0.36)

$= 7.056$ m — 5 10

(iii) At P $\vec{v} = 9\vec{i} + (14.7 - 9.8\text{x}0.6)\vec{j}$

$= 9\vec{i} + 8.82\vec{j}$ or 12.6 m/s — 5 5

(b) (i) When the particle strikes the plane at right angles

$\vec{j}$ component of r = 0 $\Rightarrow$ $20\sin\Theta.t - 4.9\cos 30.t^2 = 0$

$\Rightarrow$ $t = \frac{80\sin\Theta}{g\sqrt{3}}$ — 5

$\vec{i}$ component of v = 0 $\Rightarrow$ $20\cos\Theta - g\sin 30.t = 0$

$\Rightarrow$ $t = \frac{40\cos\Theta}{g}$ — 5

$\therefore$ $\frac{80\sin\Theta}{g\sqrt{3}} = \frac{40\cos\Theta}{g}$ $\Rightarrow$ $\tan\Theta = \frac{\sqrt{3}}{2}$ — 5 15

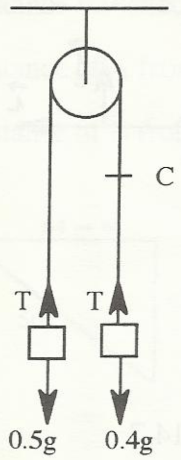
$\Theta = 40.9°$

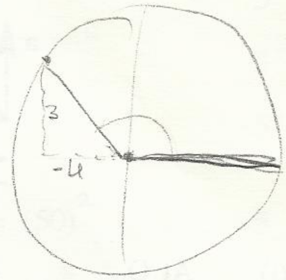
(ii) If $\Theta = 45°$ then the $\vec{i}$ component of v = 20(0.7071) − 9.8(0.5)(3.3) — 5

−2.02 < 0 — 5 10

1995

4





(i) $0.5g - T = 0.5f$ 5

$T - 0.4g = 0.4f$ 5

$0.1g = 0.9f$ 5

$f = g/9$ 5 20

(ii) velocity of A before it picks up C

$v = u + ft \Rightarrow v = 0 + \frac{g}{9}.(1)$

$\Rightarrow v = g/9$ 5 5

(iii) velocity of A after it picks up C

momentum before = momentum after

$(0.9).\frac{g}{9} = (1.1)v$ 5

$v = g/11$ 5 10

(iv) $0.5g - T = 0.5f$

$T - 0.6g = 0.6f$ 5

$-0.1g = 1.1f$

$f = -g/11$ 5

Find position of instantaneous rest

$v^2 = u^2 + 2fs \Rightarrow 0 = (g/11)^2 + 2(-g/11)s$

$\Rightarrow s = 0.45$ m above C 5 15

5(a) (i) PCM momentum before = momentum after

1995

$$2m(4u) + 3m(-u) = 2m(v_1) + 3m(v_2) \qquad 5$$

$$\Rightarrow \quad 2v_1 + 3v_2 = 5u \quad \text{........ eq1}$$

NEL $v_1 - v_2 = -e(4u + u)$ 5

$$\Rightarrow \quad v_1 - v_2 = -5eu \quad \text{........ eq2}$$

Solve equations 1 and 2

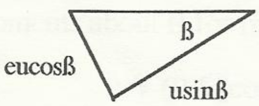
$$v_1 = u(1-3e) \qquad 5$$

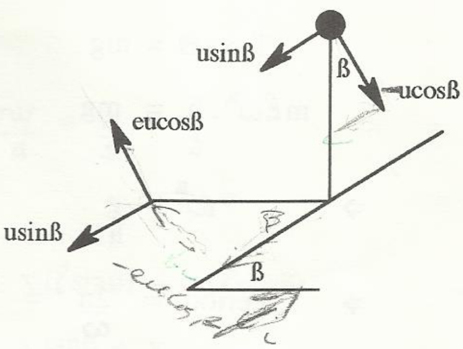
$$v_2 = u(1+2e) \qquad 5 \quad 20$$

(ii) If $e>1/3$ then $v_1 < 0$ and $v_2 > 0$ 5 5

i.e. the particles move in opposite directions after the collision.

(b) (i)





5

velocity of ball after collision along the plane = usinß

velocity of ball after collision perpendicular to the plane = eucosß 5

$$\Rightarrow \quad \tan ß = \frac{e u \cos ß}{u \sin ß}$$

$$\Rightarrow \quad \tan ß = \sqrt{e} \qquad 5 \quad 15$$

(ii) Kinetic Energy before = $0.5mu^2$ 5

Loss in kinetic energy = $0.5m(u^2\cos^2ß - e^2u^2\cos^2ß)$

$$= 0.5mu^2\cos^2ß\,(1-e^2)$$

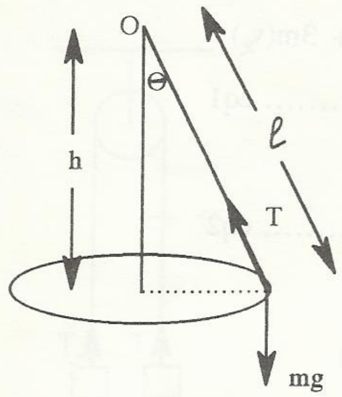
Fraction of KE lost $= \dfrac{0.5mu^2\cos^2ß\,(1-e^2)}{0.5mu^2}$

$$= \frac{(1-e)(1+e)}{(1+e)}$$

$$= 1-e \qquad 5 \quad 10$$

1995

6(a)



T , mg 5

horiz: $T \sin\Theta = mr\omega^2$ 5

$\Rightarrow \quad T\frac{r}{\ell} = mr\omega^2$

$\Rightarrow \quad T = m\ell\omega^2$

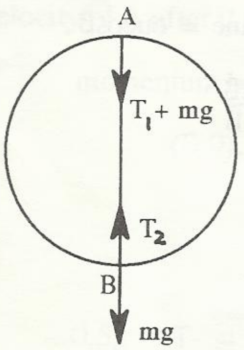
vert: $T\cos\Theta = mg$ 5

$m\ell\omega^2 . \frac{h}{\ell} = mg$

$\Rightarrow \quad \omega^2 = \frac{g}{h}$ 5

$\Rightarrow \quad \text{Period} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{g}}$ 5 25

(b)



$T_1 + mg = mr\omega_1^2$ 5

$T_2 - mg = mr\omega_2^2$ 5

Energy at A = Energy at B

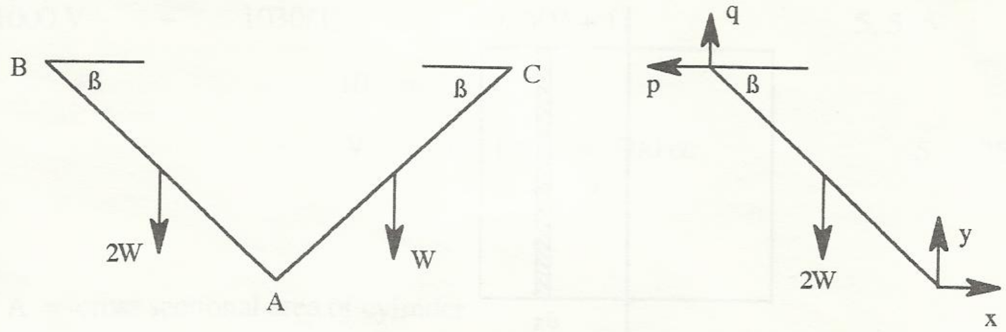
$0.5mr^2\omega_1^2 + mg(2r) = 0.5mr^2\omega_2^2$ 5, 5

$0.5(T_1 + mg) + 2mg = 0.5(T_2 - mg)$

$\Rightarrow \quad T_2 = T_1 + 6mg$ 5 25

1995

7



(i) Moments about C for the system

$$q\,(2\ell\cos\beta) = W\,(0.5\ell\cos\beta) + 2W\,(1.5\ell\cos\beta)$$ 5, 5

$$\Rightarrow \quad q = \frac{7W}{4}$$ 5

Resolve vertically for rod AB

$$q + y = 2W$$ 5

$$\Rightarrow \quad y = 2W - \frac{7W}{4} = \frac{W}{4}$$ 5

Moments about B for rod AB

$$2W\,(0.5\ell\cos\beta) = x\,(\ell\sin\beta) + y\,(\ell\cos\beta)$$ 5, 5

$$\Rightarrow \quad W = x\tan\beta + y$$

$$\Rightarrow \quad x = \frac{3W}{4\tan\beta}$$ 5 40

(ii) Reaction at B $= -\dfrac{3W}{4\tan\beta}\,\vec{i} + \dfrac{7W}{4}\,\vec{j}$

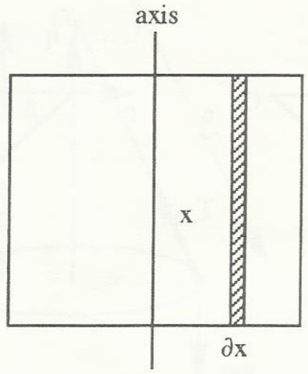
Reaction at C $= \dfrac{3W}{4\tan\beta}\,\vec{i} + \dfrac{5W}{4}\,\vec{j}$ 5

If the reactions are perpendicular then

$$\left(-\frac{3W}{4\tan\beta}\right)\left(\frac{3W}{4\tan\beta}\right) + \left(\frac{7W}{4}\right)\left(\frac{5W}{4}\right) = 0$$

$$\Rightarrow \quad \tan\beta = \frac{3}{\sqrt{35}}$$ 5 10

8(a) 1995



Let m = mass per unit area

mass of element = m(2a)dx

moment of inertia of element = $(2amdx)x^2$ 5

$$I = 2am\int_{-a}^{a} x^2 dx$$ 5

$$= 2am\left[\frac{x^3}{3}\right]_{-a}^{a}$$ 5

$$= \frac{4a^4 m}{3}$$

$$= \frac{1}{3}Ma^2 \quad \text{where } M = 4a^2 m$$ 5 20

(b) (i)

$$I = \frac{2}{3}ma^2 + mx^2$$ 5

$$Mh = mx$$ 5

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$ 5

$$= 2\pi\sqrt{\frac{\frac{2}{3}a^2 + x^2}{gx}}$$ 5 20

(ii)

$$\frac{dT}{dx} = 2\pi.\frac{1}{2}.\left(\frac{\frac{2}{3}a^2 + x^2}{gx}\right)^{-\frac{1}{2}}\left(\frac{xg.2x - (\frac{2}{3}a^2 + x^2)g}{(xg)^2}\right)$$

$$= 0$$ 5

when

$$2gx^2 = (\tfrac{2}{3}a^2 + x^2)g$$

$$2x^2 = \tfrac{2}{3}a^2 + x^2$$

$$3x^2 = 2a^2$$ 5 10

1995

9(a)		mass of water + mass of milk = mass of mixture	5	
		$1000\,V + 1030(1) = 1020(V+1)$	5, 5, 5	
		$10 = 20\,V$		
		$V = 0.5$ litre or 500 cc	5	25
(b)	(i)	Let A = cross sectional area of cylinder		
		volume immersed $= (0.185 - 0.16)A = 0.025A$		
		Total volume $= (0.1975 - 0.16)A = 0.0375A$		
		relative density of body $= \dfrac{0.025A}{0.0375A} = \dfrac{2}{3}$	5	5
	(ii)	$B = F + W$	5	
		$\dfrac{W(1)}{2/3} = F + W$		
		$F = 0.5W$		
		$= 0.5(0.02)(9.8)$		
		$= 0.098$ N	5	10
	(iii)	Find cross sectional area A		
		weight of displaced water = weight of body		
		$(1000)(0.025A)g = 0.02g$		
		$A = 0.0008\ m^2$	5	
		$\Rightarrow$ volume of water $= 0.16(0.0008)$		
		$= 0.000128\ m^3$ or 128 cc	5	10

1995

10(a) $\int y\,dy = \int \frac{4}{1+x^2}\,dx$ 5

$0.5y^2 = 4\tan^{-1}x + A$ 5

x=0, y=1 $\Rightarrow$ $0.5 = 4(0) + A$ 5

$\therefore\ 0.5y^2 = 4\tan^{-1}x + 0.5$ 5 20

or $y^2 = 8\tan^{-1}x + 1$

1995 (b) (i) $m\frac{dv}{dt} = mg - mkv$

$\int \frac{dv}{g-kv} = \int dt$ 5

$-\frac{1}{k}\ln(g-kv) = t + A$ 5

$v = 0, t = 0 \Rightarrow -\frac{1}{k}\ln(g) = A$ 5

$\therefore\ -\frac{1}{k}\ln(g-kv) + \frac{1}{k}\ln(g) = t$ 5

$\Rightarrow\ t = \frac{1}{k}\ln\left(\frac{g}{g-kv}\right)$ eq1

when $v = \frac{g}{2k}$ $t = \frac{1}{k}\ln 2$ 5 25

(ii) From equation 1

$\frac{g}{g-kv} = e^{kt}$

$g - kv = g\,e^{-kt}$

$v = \frac{g}{k}(1 - e^{-kt})$

$\rightarrow \frac{g}{k}$ as $t \rightarrow \infty$ 5 5